Bayes Theorem Notes

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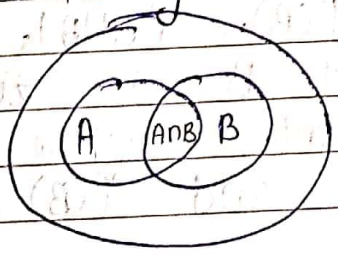
1. Conditional Probability

Conditional Probability:-

- Conditional Probability is known as the Possibility of an event or Outcome happening, based on the existence of a previous event or Outcome.
- Conditional Probability P(A|B) indicates the Probability of event 'A' happening given that event B happened.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Since B has already happened, the sample space reduces, so the Probability of A happening becomes P(A∩B) divided by P(B).



Example:- In a Batch, there are 80% C Programmers, and 40% are Java and C Programmers. What is probability that a C Programmer is also Java Programmer?

Sol:- Let A → Event, Student is Java Programmer
Let B → " " " C Programmer

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.4}{0.8} = 0.5$$

Bayes Theorem

- Bayes' theorem is also known as the formula for the probability of "causes".
- Bayes' theorem describes the probability of occurrence of an event related to any condition.

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

Likelihood, Prior, Posterior, Marginal

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(B|A) = \frac{P(B \cap A)}{P(B)}$$

$$P(A \cap B) = P(A|B) \cdot P(B) = P(B|A) \cdot P(A)$$

Theorem → If A and B are two mutually exclusive events, where P(A) is the probability of A and P(B) is the probability of B.

P(A|B) is the Probability of A given that B is true.

P(B|A) is the Probability of B given that A is true..

Then Bayes' Theorem states -

$$P(B|A) = \frac{P(B)\,P(A|B)}{P(A)\,P(B|A) + P(B)\,P(A|B)}$$

Example :- A factory has 100 machines, machine I produce 30% of the items of Output and machine II produces 70% of the items futher 5% of the items produced by the machine I were defective, and only 1% produced by machine II were defective.
If the defective item is drawn at random, What is probability that it was produced by machine I?

Sol.

A (I) = 30% Defective (D) = 5%
B (II) = 70% " (D) = 1%

Formula

$$P(A/D) = \frac{P(A)\,P(D|A)}{P(A)\,P(D|A) + P(B)\,P(D|B)}$$

A (I) → 30% $P(A) = \frac{30}{100}$

B (II) → 70% $P(B) = \frac{70}{100}$

$P(D|A) = \frac{5}{100}$

$P(D|B) = \frac{1}{100}$

$$P(A|D) = \frac{P(A)\,P(D|A)}{P(A)\,P(D|A) + P(B)\,P(D|B)}$$

$$= \frac{\frac{30}{100} \times \frac{5}{100}}{\frac{30}{100} \times \frac{5}{100} + \frac{70}{100} \times \frac{1}{100}}$$

$$= \frac{\frac{150}{10000}}{\frac{150}{10000} + \frac{70}{10000}}$$

$$= \frac{\frac{150}{10000}}{\frac{220}{10000}} \qquad \frac{150}{220} = \frac{15}{22}$$